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Determining Indentation Toughness by Incorporating True Hardness Into Fracture Mechanics Equations

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Abstract

The Lawn–Evans–Marshall (LEM) indentation fracture mechanics model for the well-developed half-penny crack is reexamined theoretically, with a particular emphasis on the physical meaning of the hardness parameter in this model. It is shown that there may be some confusion arising from the use of the hardness definition. A modified indentation toughness equation is then proposed, in which a true hardness number, rather than the apparent hardness, is incorporated. This modified equation predicts that it is the quantity $a^2/c^{3/2}$, rather than $P/c^{3/2}$, that would keep constant in the half-penny crack regime. This prediction is verified by analyzing the previously published indentation data. It is also confirmed that a comparable value of indentation toughness can be deduced with this modified equation. © 1999 Elsevier Science Limited. All rights reserved

Keywords: fracture, mechanical behaviour, hardness, indentation, modeling.

1 Introduction

Since the work of Palmqvist in 1957,[1] in which the extent of the surface radial cracking due to a sharp indentation was first explicitly recognized as being indicative of the fracture toughness of the material, the Vickers indentation test has been widely adopted for fracture toughness determination for brittle materials, particularly glasses and ceramics. Consequently, numerous semi-empirical equations relating material fracture toughness to the measured indentation parameters, such as the applied indentation load, final radial crack length, or indentation impression size, have been derived based on experimental observations and/or theoretical considerations, see for example Refs 2 and 3 and references in those sources. These semi-empirical equations can be classified into three groups. The first group is based on an assumption that the cracks which form as a result of Vickers indentation are well developed radial/median cracks [Fig. 1(a)], sometimes referred to as half-penny cracks.[4–7] The second group is based on an assumption that radial Palmqvist cracks [Fig. 1(b)] are formed.[8–11] In the third group, semi-empirical equations are established directly from the regression analysis of the indentation crack data and the fracture toughness data for a number of materials.[12–14]

However, use of the Vickers indentation test to determine fracture toughness is still problematic.[2,3] The discrepancy between the indentation fracture toughness of a material and its fracture toughness as measured by conventional methods, such as the single edge notched beam (SENB) method and the double-torsion (DT) method, has been reported frequently.[15–18] This discrepancy has been considered on the basis of a variety of phenomena, including: (i) the dependence of the crack geometry on the applied indentation load and the properties of the test material;[8,14,19] (ii) the effects of some non-ideal indentation deformation/fracture behavior, such as lateral cracking,[20] subcritical growth of indentation cracks,[21] or phase-transformation due to indentation,[22,23] and (iii) unsuitable consideration of the effects of Poisson's ratio[14] and hardness.[17]

As examined by Ponton and Rawlings,[2] hardness has been used almost without exception as an important parameter in the existing indentation fracture toughness equations. Note that the hardness used in these equations has usually been defined as the ratio of the applied indentation load to the contact or the projected area of the resulted indentation impression. For the measurement of the hardness based on such a definition, there exists

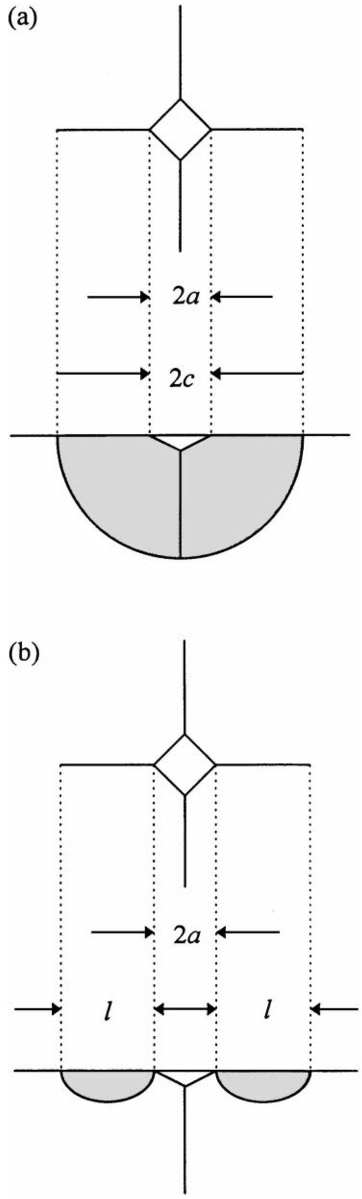

Fig. 1. Comparison of (a) median/radial and (b) Palmqvist cracks around a Vickers indentation.

a dependence of the apparent hardness on the applied indentation load, which has been well-known as the *indentation size effect*, frequently abbreviated as ISE.[24–27] The existence of ISE makes it insufficient to quote a single hardness number when hardness is used in material characterization.[27] Therefore, one can expect that uncertainties in the measured indentation fracture toughness may result from the use of a load-dependent hardness number.

The effect of the load-dependence of the apparent hardness on the determination of indentation toughness in the Palmqvist crack regime has been studied by Li *et al.*[17] It was shown that a comparable value of indentation toughness may be obtained by incorporating a load-independent hardness number, evaluated empirically, into the fracture mechanics equations for the Palmqvist cracks.[17] In this paper, a similar study is conducted in the half-penny crack regime. First, the widely cited indentation fracture mechanics model for the well-developed half-penny crack, which was proposed firstly by Lawn, Evans and Marshall (LEM model),[5] is reexamined to establish a modified indentation toughness equation, in which a true hardness number, rather than the apparent hardness, is incorporated. Then, the previously published indentation data for a soda-lime glass[28] and three grades of alumina[29] are analyzed to provide an experimental verification for the modified equation.

2 Analysis of the LEM Model

Among the existing indentation fracture mechanics models for the well-developed half-penny crack, the Lawn–Evans–Marshall (LEM) model[5] has the feature that the complex elastic–plastic indentation stress field is resolved into a reversible elastic component and an irreversible residual component. The elastic component is taken to operate outside the plastic zone, reaching its maximum intensity on full loading and reversing completely on unloading. The residual component is derived from the wedging action of the deformation zone, also reaching its maximum at full loading but persisting as the indenter is removed, providing the primary driving force for the half-penny crack configuration in the final stage.

The essence of the LEM model is that the residual stress due to indentation can be regarded as being concentrated at a point located at the crack center at the elastic-plastic interface, acting as a crack mouth opening point-force. Furthermore, it is assumed that the volume of the indentation plastic zone can be equated to that of an internally pressurized spherical cavity, allowing the use of Hill's solution to the expending spherical cavity problem.[30] Based on these assumptions, Lawn *et al.*[5] derived the following equation for the determination of fracture toughness by Vickers indentation:

$$K_C = \delta \left(\frac{E}{H_0} \right)^{1/2} \frac{P}{c^{3/2}} \qquad (1)$$

where P is the applied indentation load, c is the half-length of the half-penny crack, E is Young's modulus, H_0 is the *apparent* hardness defined as the ratio of the applied indentation load to the *projected* area of the resulting indentation impression, and δ is a non-dimensional constant that is primarily a function of the indenter geometry. The constant δ is generally regarded as a material-independent constant and its value for the standard Vickers indenter has been established by calibration with known fracture toughness values on a variety of ceramics. For example, Lawn *et al.*[5] obtained $\delta = 0{\cdot}014$; Anstis *et al.*[6] obtained $\delta = 0{\cdot}016 \pm 0{\cdot}004$. A more detailed theoretical analysis by Shetty *et al.*[3] gave $\delta = 0{\cdot}023$.

It is instructive to extend the analysis of Lawn *et al.*[5] to show how the hardness is incorporated into eqn (1). Since the original analysis did not provide an explicit expression for the constant δ, the following discussion is based on the more detailed analysis of the LEM model by Shetty *et al.*[31]

For a half-penny surface crack centrally loaded with a point force, F, the stress intensity factor is given by:[32]

$$K_I = \frac{\Omega F}{(\pi c)^{3/2}} \tag{2}$$

where c is the crack half-length, Ω is a free-surface correction factor. With the empirical equations given by Newman and Raju,[33] Ω can be calculated to be 1·2.

For the indentation problem considered here, F is the residual force derived from the indentation plastic zone. Lawn *et al.*[5] suggested the following scheme to derive F. First, due to the mismatch between the plastic zone and the surrounding elastic matrix, a residual pressure, σ_r develops at the elastic-plastic interface. If ΔV is the volume of the indentation impression and V is the volume of the indentation plastic zone, the residual pressure is given by:[31]

$$\sigma_r = \frac{E}{3(1-2\nu)}\frac{\Delta V}{V} \tag{3}$$

where ν is Poisson's ratio.

Assuming the indentation plastic zone to be hemi-spherical,[5] for Vickers indentation:

$$\Delta V = \frac{\sqrt{2}}{3\tan\varphi}a^3 \tag{4}$$

$$V = \frac{2}{3}\pi\rho^3 \tag{5}$$

where a is the half-length of the indentation diagonal, $2\varphi = 136°$ is the apex angle of the Vickers indenter, and ρ is the radius of the hemi-spherical indentation plastic zone.

Then the point force, F, is calculated by integrating the component of residual pressure, σ_r on the hemi-spherical plastic zone surface in a direction perpendicular to the crack plane:[31]

$$F = \int_0^{\pi/2} \pi\rho^2\sigma_r \sin\theta\cos\theta \, d\theta = \frac{\pi\rho^2\sigma_r}{2} \tag{6}$$

In general, it is rather difficult to measure the indentation plastic zone radius, ρ. Therefore, the relation between this radius and the elastic properties (Young's modulus, E, and Poisson's ratio, ν) and plastic properties (hardness, H) of the indented material has been a subject of many theoretical or experimental studies.[5,7,34,35] Here we use the empirical relation suggested by Lawn *et al.*[5] that approximates the more rigorous elastic-plastic solution proposed by Chiang *et al.*:[34]

$$\rho = \left(\frac{E}{H}\right)^{1/2} \frac{a}{\left(\sqrt{2}\rho\tan\varphi\right)^{1/3}} \tag{7}$$

Substituting eqns (3)–(7), together with $\varphi = 68°$, $\Omega = 1{\cdot}2$, and $\nu = 0{\cdot}25$, into eqn (2) and denoting c as the crack dimension appropriate to the well-developed equilibrium half-penny crack configuration, we obtain:

$$K_C = 0{\cdot}046(EH)^{1/2}\left(\frac{a^2}{c^{3/2}}\right) \tag{8}$$

In the LEM model, eqn (8) was simplified further as eqn (1) with a theoretical δ-value of 0·023 by defining the hardness as the ratio of the applied indentation load, P, to the *projected* area of the resulting indentation impression, $2a^2$.[31] However, it should be pointed out that such a simplification may be questioned. Equation (7) was derived directly from Hill's solution to the expanding spherical cavity problem.[5,34] According to Hill's analysis,[30] the hardness parameter used in eqn (7) should be a material constant, which is a measure of the material resistance to plastic flow, whereas the hardness defined by Lawn *et al.*[5] varies due to the existence of the ISE, as mentioned above. Indirect evidence for the existence of the difference between the hardness used in Hill's solution,[30] namely *true hardness*, H, and that used by Lawn *et al.*,[5] the *apparent hardness*, H_0, can be provided by comparing the δ-value obtained by experimental calibration, 0·014[5] or 0·016[6] with that predicted theoretically, 0·023.[31] Rewriting eqn (1) as:

$$K_C = 2\delta(EH_0)^{1/2}\left(\frac{a^2}{c^{3/2}}\right) \tag{9}$$

Note that the apparent hardness, H_0, is generally larger than the true hardness, H. The difference between the δ-values obtained by experimental calibration and that from theoretical analysis may be considered as a result of the difference between H and H_0, i.e. the higher H_0-value compared with H-value would result in a lower δ-value.

Based on the above discussion, it can be concluded that there may be some confusion arising from the use of the hardness definition in the LEM model. Consequently, the indentation toughness equation derived from the LEM model, eqn (1), should be modified as eqn (8).

3 Experimental Verifications

3.1 Dependence of indentation fracture/deformation parameter on indentation load

The preceding analysis predicts that it is the quantity $a^2/c^{3/2}$, rather than $P/c^{3/2}$, that would keep constant in the half-penny crack regime. This prediction should be examined more closely, for it has been widely accepted since the establishment of the indentation fracture mechanics[5,36–38] that a constant value of $P/c^{3/2}$ may be used as an empirical criterion for judging whether a half-penny crack configuration is well-developed during indentation.[6,16,17,19]

Let us examine the previously published indentation data for soda-lime glass[28] first. Soda-lime glass, because of its transparency, homogeneity, and general availability, was usually used as a reference material in the previously cited indentation fracture mechanics studies. The evolution of the half-penny crack configuration and the extension of the so-called hemispherical plastic zone beneath the indentation impression can be easily followed in this material. Recently, Maschio and Nobile[28] performed Vickers indentation experiments in standard procedure at various maximum loads on soda-lime glass specimens; the original data were listed completely in Table 1 of Ref. 28. These original data are now illustrated in Fig. 2, where the values of both quantities, $a^2/c^{3/2}$ and $p/c^{3/2}$, are plotted as functions of the applied indentation load, P. A decreasing tendency in $p/c^{3/2}$ with the indentation load is evident, while the quantity $a^2/c^{3/2}$ is effectively invariant with respect to load. Furthermore, the coefficients of variation of $a/c^{3/2}$ and $P/c^{3/2}$ are calculated to be 0·06 and 0·18, respectively, giving an indirect support for the theoretical prediction mentioned above.

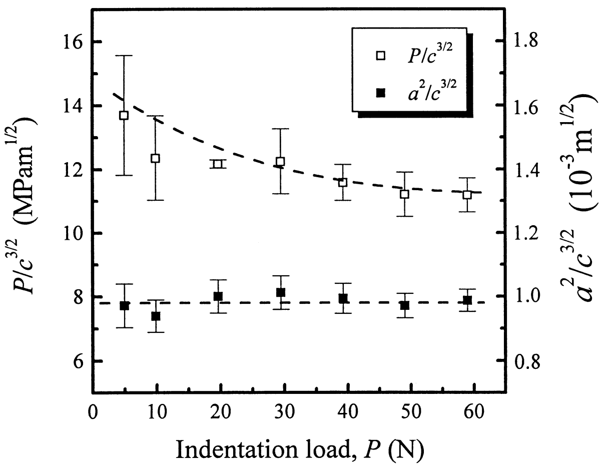

Fig. 2. Plots of $a^2/c^{3/2}$ and $P/c^{3/2}$ over the working range of load P for soda-lime glass with the experimental data reported by Maschio and Nobile.[28]

Similar conclusions can also be obtained by analyzing the experimental results reported by Franco *et al.*[29] Three grades of alumina ceramics with different grain sizes G, $G = 1{\cdot}2\,\mu$m for sample F, $3{\cdot}8\,\mu$m for sample M, and $14{\cdot}1\,\mu$m for sample C, were studied by Franco *et al.*[29] Vickers indentation tests were performed on these materials with loads ranging from 4·91 to 245·3 N. The experimental results are shown in Fig. 3. For samples F and M, $a/c^{3/2}$ keeps nearly constant whereas $P/c^{3/2}$ decreases as indentation load increases. For sample C, a slight increases in $a^2/c^{3/2}$, as well as $P/c^{3/2}$, exists in the higher load range. This may be due to the fact that the coarse-grained alumina usually exhibits a rising crack-growth-resistance behavior.[39,40]

An extensive study of the applicability of eqn (1), which was derived from the LEM model,[5] to determine the fracture toughness of ceramics has been conducted by Anstis *et al.*[6] In this study, a number of reference materials, including glasses, a glass-ceramic and polycrystalline ceramics, were indented with a Vickers indenter to determine the crack size, c, as a function of applied indentation load, P. It was suggested from the plots of $P/c^{3/2}$ against P that $P/c^{3/2}$ was independent of P for each material with experimental scatter. If the experimental results of Anstis *et al.* shown in Fig. 4 of Ref.6 are closely examined, however, a decreasing tendency in the measured $P/c^{3/2}$ with P can also be detected, especially for the results on Si_3N_4 (NC350), Al_2O_3 (AD999), glass (LA) and Si.

Table 1. Comparisons of the K_C-values obtained with different methods

Material	*Soda-lime glass*	*Alumina (sample F)*	*Alumina (sample M)*
Elastic modulus E (GPa)	70	390	390
True hardness H (GPa)	4·5	19	18
K_C (MPa$\sqrt{m}$)			
Present method ($K_C{}^M$)	0·80 ± 0·02	3·82 ± 0·09	3·85 ± 0·10
LEM method[a] ($K^L{}_C$)	0·67 ± 0·02	2·97 ± 0·12	3·01 ± 0·08
Other method ($K^R{}_C$)	0·74[b]	3·66[c]	3·55[c]

[a]Calculated with a δ-value of 0·016.
[b]With double canti-level beam (BCD) method.[6]
[c]From Hertzian tests.[29]

3.2 Indentation toughness determination

In order to assess the accuracy and the efficiency of the modified indentation toughness equation, eqn (8), the previously published data for soda-lime glass[28] and two grades of alumina ceramics, samples F and M studied by Franco *et al.*,[29] are quoted again to conduct the K_C calculation. Because of the existence of a rising crack-growth-resistance behavior, sample C used by Franco *et al.*[29] is not considered here.

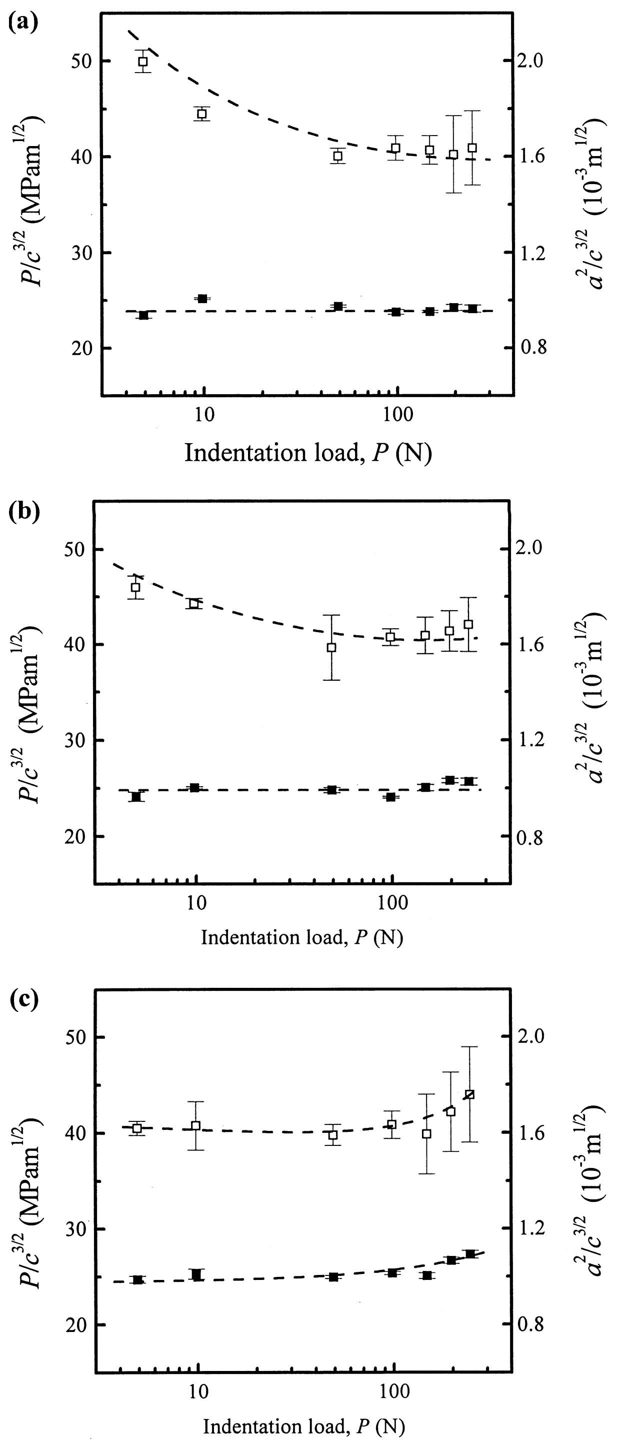

Fig. 3. Plots of $a^2/c^{3/2}$ (□) and $p/c^{3/2}$ (■) over the working range of load P for alumina ceramics with the experimental data reported by Franco *et al.*[29] (a) $G = 1{\cdot}2\ \mu$m; (b) $G = 3{\cdot}8\ \mu$m; (c) $G = 14{\cdot}1\ \mu$m

According to the preceding analysis, the determination of indentation toughness with the modified equation, eqn (8), requires prior knowledge of the true hardness of the test material. Recently, Li and Bradt[26] proposed a 'Proportional Specimen Resistance' (PSR) model to describe the indentation size effect on the apparent hardness. In this model, the applied indentation load, P, and the resulted indentation impression size, $2a$, were found to follow the relationship:[26]

$$P = \alpha_1(2\mathrm{a}) + \alpha_2(2\mathrm{a})^2 \qquad (10)$$

where α_1 and α_2 are constants. Especially, α_2 in eqn (10) was suggested to be related to the load-independent hardness, or true hardness H, by the following equation:

$$H = \kappa\alpha_2 \qquad (11)$$

where κ is a constant dependent only on the indenter geometry. For the standard Vickers indenter, $\kappa = 1{\cdot}8544$.

The applicability of eqn (10) to describe the ISE over a relatively wide range of applied indentation load has been explored by Gong *et al.*[41,42] It was found that eqn (10) should be modified as:

$$P = \alpha_0 + \alpha_1(2\mathrm{a}) + \alpha_2(2\mathrm{a})^2 \qquad (12)$$

where α_0 is a constant and the physical meaning of α_2 is the same as that in eqn (10).

Equations (11) and (12) are now chosen to determine the true hardness number for soda-lime glass. In Ref. 28 the indentation tests on soda-lime glass were performed with two kinds of pyramidal indenters of different apex angles, one being the standard Vickers indenter with the apex angle of

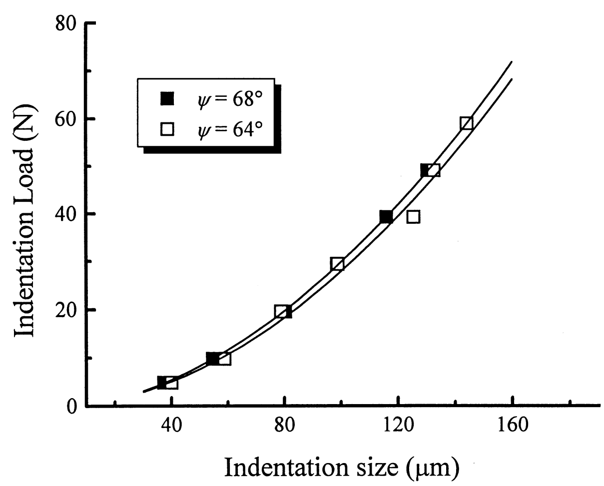

Fig. 4. P–d relations measured with different indenters for soda-lime glass.

136° and the other being a non-standard indenter with an apex angle of 128°. This makes it possible to evaluate the reliability of the resulting true hardness number. The original data listed in Table 1 of Ref. 28 for both indenter geometries are plotted in Fig. 4. The solid lines in this plot are obtained by a conventional polynomial regression according to eqn (12). Clearly, eqn (12) is proven to be sufficiently suitable for the representation of the experimental data. The α_2-value is determined to be 2·44 GPa for the standard indenter and 2·47 GPa for the non-standard indenter. Accordingly, the true hardness of soda-lime glass can be calculated with eqn (11) to be 4·52 GPa (with $\kappa = 1{\cdot}8544$) and 4·44 GPa (with $\kappa = 1{\cdot}7976$), respectively. The true hardness values obtained with the two different indenter geometries are in good agreement with each other, implying that the modified PSR model, eqn (12), seems to be an effective approach to determine the true hardness number.

The true hardness numbers of the two grades of alumina ceramics, samples F and M, can also be determined with the same procedure described above. Figure 5 shows the polynomial regression

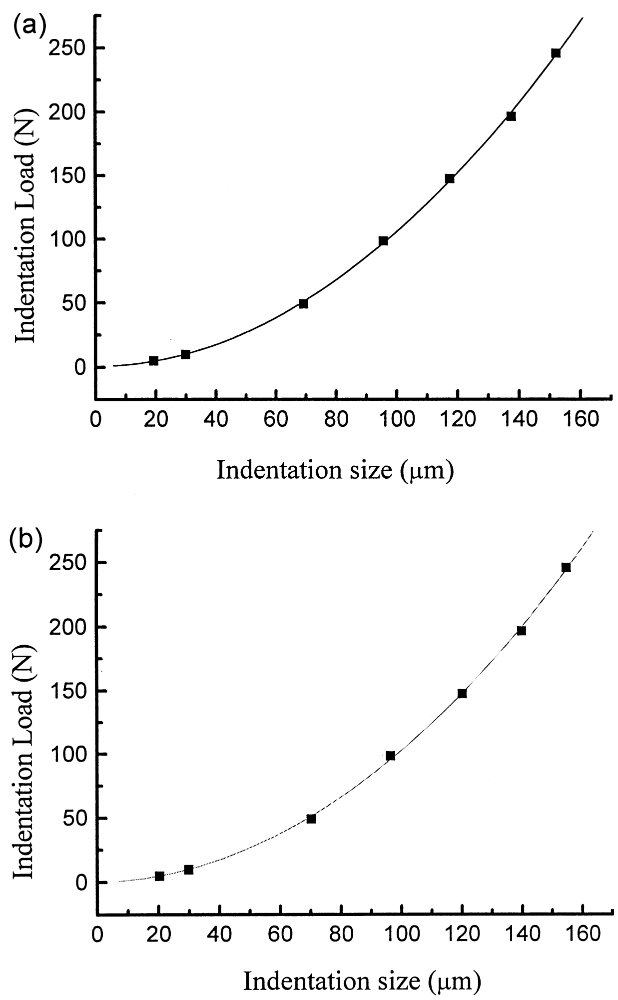

Fig. 5. *P–d* relations measured for alumina ceramics: (a) $G = 1{\cdot}2\,\mu$m; (b) $G = 3{\cdot}8\,\mu$m.

analysis results for these two samples and the resulting true hardness values are given in Table 1.

Having determined the true hardness numbers, it is now possible to conduct the K_C calculations for all samples considered with the modified method, eqn (8). The calculated results, denoted as $K_C{}^M$, are given in Table 1. Also listed in Table 1 are the K_C-values obtained with the original LEM method (denoted as $K_C{}^L$) and other methods (denoted as $K_C{}^R$). For each sample, both $K_C{}^M$ and $K_C{}^R$ are nearly identical with each other, indicating that a comparable K_C-value can be obtained with the present modified method and giving a convincing support for the theoretical analysis presented in Section 2.

It is of interest to make a brief comment on the K_C-values obtained with the original LEM method, eqn (1). As can be seen in Table 1, the original LEM method gives a reasonable K_C-value for soda-lime glass but underestimates significantly the toughness for both alumina ceramics. This can be understood easily by noting that the δ-value used for $K_C{}^L$ calculation with eqn (1), $\delta = 0{\cdot}016$, is only an experimentally calibrated constant. Assuming that the indentation parameter $P/c^{3/2}$ is effectively invariant with respect to indentation load for a given material, Anstis *et al.*[6] performed Vickers indentation experiments on a series of so-called 'reference' materials whose $K_C{}^R$-values were predetermined independently by other conventional methods, and calculated the δ-value for each material with the resulting $P/c^{3/2}$ value from the following equation:

$$\delta = K_C{}^R \left(\frac{H_0}{E}\right)^{1/2} \left(\frac{P}{c^{3/2}}\right) \tag{13}$$

The widely cited δ-value, $\delta = 0{\cdot}016$, was then deduced by averaging over the data for all 'reference' materials tested. Since the coefficient of variation of this calibrated δ-value is too high, about 25%,[6] to ensure an enough accuracy in the $K_C{}^L$ calculation with eqn (1) for any material, some systematic discrepancies may be expected between the resulting $K_C{}^L$-value and the corresponding $K_C{}^R$-value, at least in some certain cases. Clearly, the large scatter in the δ-values calculated with eqn (13) can be attributed to the fact that both H_0 and $P/c^{3/2}$ are unreasonably treated as constants.

4 Summary and Conclusions

A detailed theoretical analysis has shown that there may be potential for error arising from the use of the hardness definition in the Lawn–Evans–Marshall

(LEM) model. Accordingly, the indentation toughness equation derived from the LEM model has been modified. This modified equation predicts that it is the quantity, $a^2/c^{3/2}$, rather than $P/c^{3/2}$, that is constant in the half-penny crack regime. The previously published indentation data for some ceramics have been reexamined, indicating that the quantity $a^2/c^{3/2}$ is effectively invariant with respect to load, and giving a convincing support for the theoretical predictions.

Determining indentation toughness with the modified equation proposed in this study requires prior knowledge of the true hardness of the test material. An empirical method to determine the true hardness is suggested. The hardness number determined with this empirical method is independent of indenter geometry as well as of applied indentation load. If such a true hardness number is incorporated within the calculation, the toughness value determined with the modified equation is in agreement with that determined by conventional measurement methods.

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